

## Enhanced spin Hall effect in semiconductor heterostructures with artificial potential

Mikio ETO\* and Tomohiro YOKOYAMA

Faculty of Science and Technology, Keio University, 3-14-1 Hiyoshi, Kohoku-ku, Yokohama 223-8522, Japan

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We theoretically investigate an extrinsic spin Hall effect (SHE) in semiconductor heterostructures due to the scattering by an artificial potential created by antidot, STM tip, etc. The potential is electrically tunable. First, we formulate the SHE in terms of phase shifts in the partial wave expansion for two-dimensional electron gas. The effect is significantly enhanced by the resonant scattering when the attractive potential is properly tuned. Second, we examine a three-terminal device including an antidot, which possibly produces a spin current with polarization of more than 50%.

KEYWORDS: spin Hall effect, spin-orbit interaction, antidot, STM, resonant scattering, spin filter

The spin-orbit (SO) interaction in semiconductors has attracted a lot of attention for its possible application to manipulate electron spins in spin-based electronics, “spintronics.”<sup>1)</sup> The SO interaction is a relativistic effect and written as

$$H_{\text{SO}} = \frac{\lambda}{\hbar} \boldsymbol{\sigma} \cdot [\mathbf{p} \times \nabla V(\mathbf{r})] \quad (1)$$

for electrons in the vacuum, where  $V(\mathbf{r})$  is an external potential and  $\boldsymbol{\sigma}$  indicates the electron spin  $\mathbf{s} = \boldsymbol{\sigma}/2$ . The coupling constant is given by  $\lambda = -\hbar^2/(4m_0^2c^2)$  with electron mass  $m_0$  and velocity of light  $c$ . For conduction electrons in direct-gap semiconductors, the SO interaction is expressed in the same form. The coupling constant  $\lambda$  is much larger than the value in the vacuum owing to the band effect, particularly in narrow-gap semiconductors such as InAs.<sup>2)</sup>

A well-known example is the Rashba SO interaction in two-dimensional electron gas (2DEG) in semiconductor heterostructures.<sup>3,4)</sup> An electric field perpendicular to the 2DEG in the  $xy$  plane,  $V(\mathbf{r}) = e\mathcal{E}z$ , gives rise to

$$H_{\text{SO}} = \frac{\alpha}{\hbar} (p_y \sigma_x - p_x \sigma_y), \quad (2)$$

with  $\alpha = e\mathcal{E}\lambda$ . The large values of  $\alpha$  have been reported in experiments.<sup>5-7)</sup> The spin transistor proposed by Datta and Das is based on this Rashba SO interaction because of its tunability by the external field.<sup>8)</sup> The electron spins are manipulated by the SO interaction in semiconductor, which are injected from a ferromagnet and detected by another ferromagnet.

The SO interaction may also be useful for the spin injection, instead of using ferromagnets, in the spintronics devices. The spin Hall effect (SHE) is one of the phenomena to generate a spin current. The effect is categorized into two, intrinsic and extrinsic SHEs. The former is induced by the drift motion of holes in the SO-split

valence bands<sup>9,10)</sup> or that of electrons in the conduction band in the presence of Rashba SO interaction.<sup>11)</sup> The latter stems from the impurity scattering. For centrally symmetric potential around an impurity,  $V(r)$ , eq. (1) is rewritten as

$$H_{\text{SO}} = -\lambda \frac{2}{r} \frac{dV}{dr} \mathbf{l} \cdot \mathbf{s}, \quad (3)$$

where  $\mathbf{l} = (\mathbf{r} \times \mathbf{p})/\hbar$  is the angular momentum. This results in the skew scattering: Accompanied by the scattering from direction  $\mathbf{n}$  to  $\mathbf{n}'$ , the spin is polarized in  $(\mathbf{n} \times \mathbf{n}')/|\mathbf{n} \times \mathbf{n}'|$ .<sup>12,13)</sup> In an optical experiment of Kerr rotation, Kato *et al.* have observed a spin accumulation at sample edges transverse to the electric current in  $n$ -type GaAs.<sup>14)</sup> This is ascribable to the extrinsic SHE due to the scattering of conduction electrons by the screened Coulomb potential around charged impurities.<sup>15)</sup>

In the present letter, we focus on the extrinsic SHE in 2DEG in semiconductor heterostructures. We begin with the quantum mechanical formulation of the effect. Although the extrinsic SHE is usually described by a semi-classical theory considering the skew scattering and “side jump” effects,<sup>15)</sup> the quantum theory is required to fully understand the SHE and should be useful in designing spintronics devices based on 2DEG. We stress that the SHE is easier to understand in 2DEG than in three-dimensional case. Second, we examine the SHE in 2DEG by an artificial potential created by antidot, STM tip, etc. The antidot is a small metallic electrode fabricated above the 2DEG, as schematically shown in the inset in Fig. 1, to create a scattering potential for electrons. The potential is electrically tunable and may be attractive as well as repulsive. We show that the SHE is significantly enhanced by the resonant scattering when the attractive potential is properly tuned. Finally, we propose a three-terminal device including an antidot. Until now, several spin-filtering devices have been proposed using semiconductor nanostructures with SO interaction.<sup>16-22)</sup>

\*E-mail address: eto@rk.phys.keio.ac.jp

Recently, Yamamoto and Kramer have studied a three-terminal spin filter with a repulsive antidot potential.<sup>23)</sup> We show that a similar device with an attractive antidot potential could be a spin filter with an efficiency of more than 50% by the tuning to the resonance.

We consider a scattering problem of an electron in the  $xy$  plane by an axially symmetric potential  $V(r)$  ( $r = \sqrt{x^2 + y^2}$ ). The SO interaction is given by

$$H_{SO} = -\lambda \frac{2}{r} \frac{dV}{dr} l_z s_z \equiv V_1(r) l_z s_z, \quad (4)$$

with  $l_z$  and  $s_z$  being the  $z$  component of angular momentum and spin operators.  $V_1(r) = -(2\lambda/r)V'(r)$  has the same sign as  $V(r)$  when  $|V(r)|$  is a monotonically decreasing function of  $r$  and  $\lambda > 0$ . Assuming that  $V(r)$  is smooth in the scale of lattice constant, we adopt the effective mass equation

$$\left[ -\frac{\hbar^2}{2m^*} \Delta + V(r) + V_1(r) l_z s_z \right] \psi(\mathbf{r}) = E \psi(\mathbf{r}), \quad (5)$$

for an envelope function  $\psi(\mathbf{r})$  with effective mass  $m^*$ . The Dresselhaus SO interaction is neglected, which stems from an inversion asymmetry of the crystal.<sup>24)</sup>

Note that  $l_z$  and  $s_z$  are conserved in eq. (5), in contrast to the three-dimensional case with eq. (3), which simplifies the discussion. For  $s_z = \pm 1/2$ , an electron feels the potential of  $V(r) \pm V_1(r)l_z/2$ . In consequence the scattering for components of  $l_z > 0$  ( $l_z < 0$ ) is enhanced (suppressed) by the SO interaction for  $s_z = 1/2$  when  $V_1(r)$  has the same sign as  $V(r)$ . The effect is opposite for  $s_z = -1/2$ . This is the origin of the extrinsic SHE.

We adopt a partial wave expansion for the scattering problem with  $l_z = m = 0, \pm 1, \pm 2, \dots$ .<sup>25)</sup> As an incident wave, we consider a plane wave propagating in  $x$  direction,  $e^{ikx}$ , with spin  $s_z = 1/2$  or  $-1/2$ .  $E = \hbar^2 k^2 / (2m^*)$ . The plane wave is expanded as

$$e^{ikx} = e^{ikr \cos \theta} = \sum_{m=-\infty}^{\infty} i^m J_m(kr) e^{im\theta}, \quad (6)$$

where  $\theta$  is the angle from  $x$  direction and  $J_m$  is the  $m$ th Bessel function. Its asymptotic form at  $r \rightarrow \infty$  is given by  $J_m(kr) \sim \sqrt{2/(\pi kr)} \cos(kr - m\pi/2 - \pi/4)$ . In the solution of eq. (5),  $J_m(kr)$  in eq. (6) is replaced by  $R_m^\pm(r)$  for  $s_z = \pm 1/2$ ,<sup>26)</sup> which satisfies

$$\begin{aligned} \left[ -\frac{\hbar^2}{2m^*} \left( \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} - \frac{m^2}{r^2} \right) + V(r) \pm \frac{m}{2} V_1(r) \right] R_m^\pm(r) \\ = E R_m^\pm(r). \end{aligned} \quad (7)$$

Its asymptotic form determines the phase shift  $\delta_m^\pm$ :

$$R_m^\pm(r) \sim \sqrt{\frac{2}{\pi kr}} e^{i\delta_m^\pm} \cos \left( kr - \frac{m\pi}{2} - \frac{\pi}{4} + \delta_m^\pm \right). \quad (8)$$

From eqs. (7) and (8), we immediately obtain the rela-

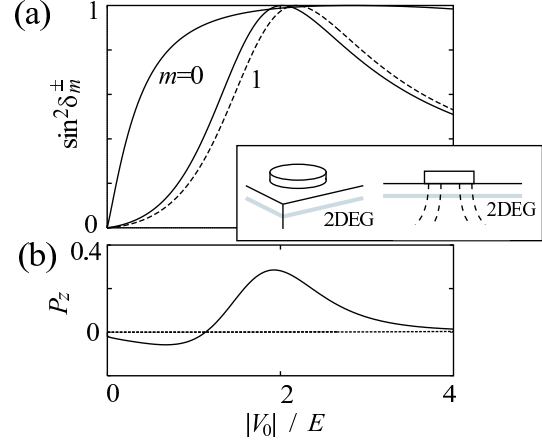


Fig. 1. Extrinsic SHE due to the scattering by a potential well,  $V(r) = V_0 \theta(a-r)$  ( $V_0 < 0$ ), for 2DEG.  $a = 1/k$ . The strength of the SO interaction is  $\lambda k^2 = 0.01$  ( $\lambda = 117.1 \text{ \AA}^2$ ,  $2\pi/k = 70 \text{ nm}$ ). (a) The scattering probability of each partial wave,  $\sin^2 \delta_m^\pm$ , and (b) spin polarization  $P_z$  at  $\theta = -\pi/2$ , as functions of the potential depth  $|V_0|$  [normalized by electron energy  $E = \hbar^2 k^2 / (2m^*)$ ]. In (a), solid and broken lines indicate the cases of  $s_z = 1/2$  and  $-1/2$ , respectively, for  $m = 1$  ( $\delta_{-1}^\pm = \delta_1^\mp$ ). The scattering probability for  $|m| \geq 2$  is negligibly small. Inset: schematic drawing of artificial potential on 2DEG created by antidot. The potential is attractive (repulsive) when a positive (negative) voltage is applied to the antidot.

tions of  $\delta_{-m}^\pm = \delta_m^\mp$ , indicating the time reversal symmetry. The SO interaction does not work for the  $S$  wave ( $m = 0$ ):  $\delta_0^+ = \delta_0^- \equiv \delta_0$ .

The scattering amplitude  $f^\pm(\theta)$  for  $s_z = \pm 1/2$  is expressed in terms of the phase shifts:  $f^\pm(\theta) = f_1(\theta) \pm f_2(\theta)$ ,

$$\begin{aligned} f_1(\theta) &= \frac{1}{i\sqrt{2\pi k}} \left[ e^{2i\delta_0} - 1 \right. \\ &\quad \left. + \sum_{m=1}^{\infty} \left( e^{2i\delta_m^+} + e^{2i\delta_m^-} - 2 \right) \cos m\theta \right], \quad (9) \end{aligned}$$

$$f_2(\theta) = \frac{1}{\sqrt{2\pi k}} \sum_{m=1}^{\infty} (e^{2i\delta_m^+} - e^{2i\delta_m^-}) \sin m\theta. \quad (10)$$

The scattering cross section is given by  $\sigma^\pm(\theta) = |f^\pm(\theta)|^2$ . Hence the spin polarization of the scattered wave in  $\theta$  direction is expressed as

$$P_z = \frac{|f^+|^2 - |f^-|^2}{|f^+|^2 + |f^-|^2} = \frac{2\text{Re}(f_1 f_2^*)}{|f_1|^2 + |f_2|^2}, \quad (11)$$

when the incident electron is unpolarized. This formula is analogous to that of skew scattering in three-dimensions,<sup>12,13)</sup> and one of the main results in the present letter. The spin is polarized in  $z$  direction and  $P_z(-\theta) = -P_z(\theta)$ .

Now we examine the SHE due to the scattering by an attractive potential. The simplest example is a potential

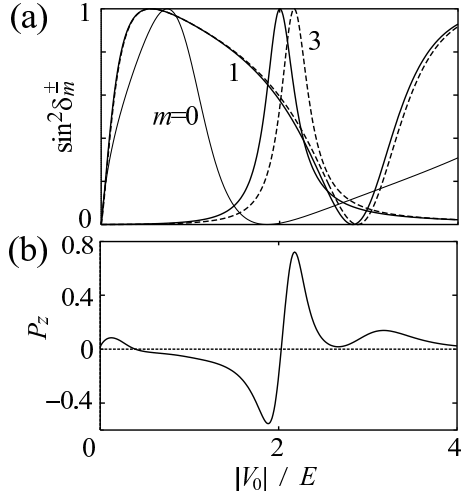


Fig. 2. Extrinsic SHE due to the scattering by a potential well,  $V(r) = V_0\theta(a-r)$  ( $V_0 < 0$ ), for 2DEG.  $a = 2/k$ . The strength of the SO interaction is  $\lambda k^2 = 0.01$ . (a) The scattering probability of each partial wave,  $\sin^2 \delta_m^\pm$ , and (b) spin polarization  $P_z$  at  $\theta = -\pi/2$ , as functions of the potential depth  $|V_0|$  [normalized by electron energy  $E = \hbar^2 k^2 / (2m^*)$ ]. In (a), solid and broken lines indicate the cases of  $s_z = 1/2$  and  $-1/2$ , respectively, for  $m > 0$  ( $\delta_{-m}^\pm = \delta_m^\mp$ ). The data for  $|m| = 2$  are omitted.

well,  $V(r) = V_0\theta(a-r)$  ( $V_0 < 0$ ,  $a > 0$ ), where  $\theta(t)$  is a step function [ $\theta(t) = 1$  for  $t > 0$ ,  $0$  for  $t < 0$ ]. Then  $V_1 = (2\lambda/a)V_0\delta(r-a)$  with  $\delta$ -function  $\delta(t)$ . The phase shifts  $\delta_m^\pm$  are calculated by solving eq. (7).

Figure 1 shows the scattering probability of each partial wave,  $\sin^2 \delta_m^\pm$ , and spin polarization  $P_z$  at  $\theta = -\pi/2$ , as functions of the potential depth  $|V_0|$ . The strength of the SO interaction is set to be  $\lambda k^2 = 0.01$ , which corresponds to the value of InAs,  $\lambda = 117.1 \text{ \AA}^2$ , with the electron wavelength  $2\pi/k = 70 \text{ nm}$ . The radius of the potential well is  $a = 1/k$ . With an increase in  $|V_0|$ , the scattering probability increases and becomes unity at some values of  $|V_0|$  (unitary limit with  $\delta_m^\pm = \pi/2$ ) for  $m = 0$  ( $S$  wave) and  $m = \pm 1$  ( $P$  wave). This is due to the resonant scattering through virtual bound states in the potential well. The resonant width is narrower for larger  $|m|$  because of the centrifugal potential  $\propto m^2/r^2$  separating the bound states from the outer region. Around the resonance of the  $P$  waves, a difference in  $\delta_1^+ - \delta_1^- \equiv \Delta\delta_1$  results in a large spin-polarization  $P_z \approx 30\%$ . Around the resonance,  $(\delta_1^+ + \delta_1^-)/2 \approx \pi/2$ , formula (11) yields

$$P_z(\theta = -\pi/2) \approx \frac{2 \sin^2 \delta_0 \sin \Delta\delta_1}{\sin^2 \delta_0 + \sin^2 \Delta\delta_1}$$

when  $\delta_m^\pm$  ( $|m| \geq 2$ ) is negligibly small.

Figure 2 shows the calculated results for a wider potential well,  $a = 2/k$ . The resonant scattering takes place for  $0 \leq |m| \leq 3$  (not shown for  $|m| = 2$  because the resonance with even  $m$  is not relevant to the spin polarization at  $\theta = -\pi/2$ ). Around the resonance of  $F$  waves

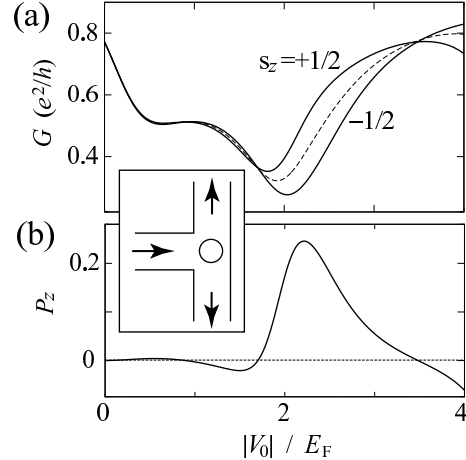


Fig. 3. Numerical results of spin injection in a three-terminal device including a tunable antidot potential, schematically shown in the inset. (a) Conductance  $G_\pm$  from the left lead to the lower lead with spin  $s_z = \pm 1/2$ , and (b) spin polarization of the current in the lower lead, as functions of the depth of attractive potential  $|V_0|$  (normalized by Fermi energy  $E_F$ ). The strength of SO interaction is  $\lambda k_F^2 = 0.028$ , with  $k_F$  being the Fermi wavenumber ( $\lambda = 117.1 \text{ \AA}^2$ ,  $2\pi/k_F = 40 \text{ nm}$ ). The radius of the potential is  $a = 2/k_F$ . In (a), broken line indicates the conductance per spin in the absence of SO interaction.

( $|m| = 3$ ),  $P_z$  is enhanced to 72%. In general, a sharper resonance enlarges  $\delta_m^+ - \delta_m^-$  for larger  $|m|$ , which results in a larger polarization.

The extrinsic SHE is expected even with a repulsive potential in the presence of SO interaction, eq. (4). We solve the scattering problem with a potential barrier,  $V(r) = V_0\theta(a-r)$  with  $V_0 > 0$  and  $a = 2/k$ . We find that the spin polarization  $P_z(\theta = -\pi/2)$  is less than 0.5% in the range of  $0 < V_0/E < 4$  (not shown), which is much smaller than the values in Fig. 2 with attractive potential. This indicates the importance of the resonant scattering for the enhancement of SHE.

Making use of a tunable antidot potential, we propose a three-terminal device as an efficient spin filter [inset in Fig. 3]. Unpolarized electrons are injected from the left lead (connected to the source electrode) and outgoing into upper and lower leads (connected to the drain electrodes). The voltages are equal in the two drains. We assume a hard-wall potential for the boundaries of leads and a smooth potential well for an antidot,  $V(r) = V_0$  ( $r < a - \Delta/2$ ),  $(V_0/2)[1 - \sin[\pi(r-a)/\Delta]]$  ( $|r-a| < \Delta/2$ ),  $0$  ( $r > a + \Delta/2$ ), where  $V_0 < 0$  and  $r$  is the distance from the center of the junction. We set  $\Delta = 0.7a$ . In the presence of SO interaction, eq. (4),  $s_z$  is conserved, whereas  $l_z$  is not owing to the lack of rotational symmetry of the system.

The conductance  $G_\pm$  for  $s_z = \pm 1/2$  is numerically evaluated in the same way as in ref. 22, using the Green function's recursion method on the tight-binding model of a square lattice ( $31 \times 31$  sites in the junction area). The

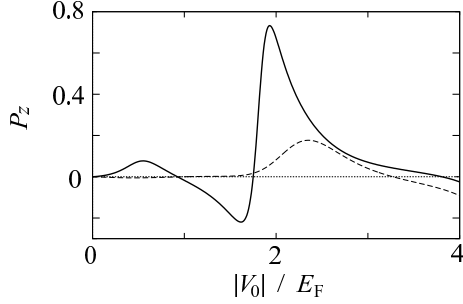


Fig. 4. Spin polarization  $P_z$  for each channel in the incident current in the three-terminal device given in Fig. 3. Solid and broken lines indicate  $P_z$  for the lowest mode and second mode, respectively. The parameters are the same as in Fig. 3.

temperature is  $T = 0$ . The spin polarization is defined as  $P_z = (G_+ - G_-)/(G_+ + G_-)$ .

Figure 3 shows the conductance  $G_{\pm}$  from the source to the lower drain, with the spin polarization  $P_z$ , as functions of the potential depth  $|V_0|$ . (The spin polarization is  $-P_z$  in the upper drain.) We assume that the Fermi wavelength is  $2\pi/k_F = 40\text{nm}$  ( $k_F$  is the Fermi wavenumber) and thus  $\lambda k_F^2 = 0.028$  with  $\lambda = 117.1\text{\AA}^2$ . The potential radius is  $a = 2/k_F$  and the width of leads is  $W = 4a = 8/k_F$ . Then there are two conduction modes in the leads at the Fermi energy  $E_F$ . The spin polarization is enhanced to 25% around  $|V_0|/E_F = 2$ , which is attributable to the resonant scattering via a virtual bound state around the antidot,<sup>27)</sup> as discussed before.

To examine the resonance in detail, we make a channel analysis for two incident modes from the left lead. In Fig. 4, we plot  $P_z$  separately for the lowest mode,  $e^{ik_1x} \cos(\pi y/W)$ , and for the second mode,  $e^{ik_2x} \sin(2\pi y/W)$  [ $k_1^2 + (\pi/W)^2 = k_2^2 + (2\pi/W)^2 = k_F^2$ ]. The lowest mode plays a main role in the enhancement of spin polarization around the resonance. Since we could selectively inject the lowest mode to the junction, e. g., using a quantum point contact fabricated on the left lead, our device could be a spin filter with an efficiency of more than 50%.

In conclusion, we have formulated the extrinsic SHE for 2DEG in semiconductor heterostructures, using the quantum mechanics. We have examined the SHE due to the scattering by a tunable potential created by antidot, STM tip, etc. The resonant scattering significantly enhances the SHE with an attractive potential. We have proposed a three-terminal device including an antidot, as an efficient spin filter.

A three-terminal spin filter without antidot has been studied by Kiselev and Kim in the presence of Rashba SO interaction.<sup>16)</sup> They have pointed out an enhancement of spin polarization by the resonant scattering at the junction when the Fermi energy of 2DEG is tuned. In their device, the direction of spin polarization is tilted

from the  $z$  direction perpendicular to the plane. In our device, the spin is polarized in  $z$  direction, which is easier to detect by the optical experiment,<sup>14)</sup> and above all, more suitable to the spintronics devices.

The extrinsic SHE enhanced by (many-body) resonant scattering has been examined for metallic systems with magnetic impurities.<sup>28–30)</sup> In the case of semiconductor heterostructures, however, we have a great advantage in the tunability of potential. The SHE by the resonant scattering at a single potential can be investigated in details.

We make some comments regarding our device. (i) The electron-electron interaction has been neglected in our calculations. The number of electrons trapped in the potential well is given by the Friedel sum rule,  $(1/\pi) \sum_m \sum_{\sigma} \delta_m^{\sigma}$ . The Hartree potential from the electrons should be considered although the Coulomb blockade is irrelevant to the antidot potential without tunnel barriers in contrast to usual quantum dots. Therefore, the values of  $|V_0|$  at the resonance have been underestimated. (ii) It is required to create such a deep potential as  $|V_0| \sim E_F$  in designing the device. (iii) We have assumed that the antidot potential  $V(\mathbf{r})$  does not depend on  $z$ . Otherwise, the Rashba-type SO interaction, eq. (2) with  $\alpha = \lambda(\partial V/\partial z)$ , has to be added to eq. (4). It would make an effective magnetic field in the  $xy$  plane and thus decrease the spin polarization in  $z$  direction.

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